# Is Mackie's superposition derivation wrong?

Presenter: Chengzhu Zhang

3/6/2024

RUTGERS
Robert Wood Johnson
Medical School

# Background

RUTGERS

- Mackie’s textbook paper on radiation dose calculation derived the superposition method.
  - The fundamental method AAA is based on.

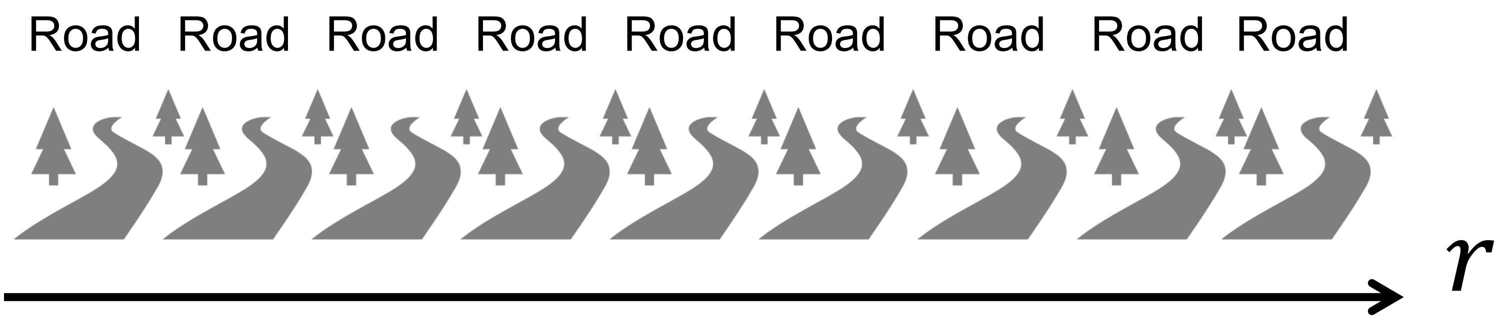
Mackie, T. R., J. W. Scrimger, and J. J. Battista.
"A convolution method of calculating dose for 15-MV x rays."
Medical physics 12.2 (1985): 188-196.

- The purpose of this presentation is to see whether we can derive his dose calculation algorithm

# An analogy to energy loss

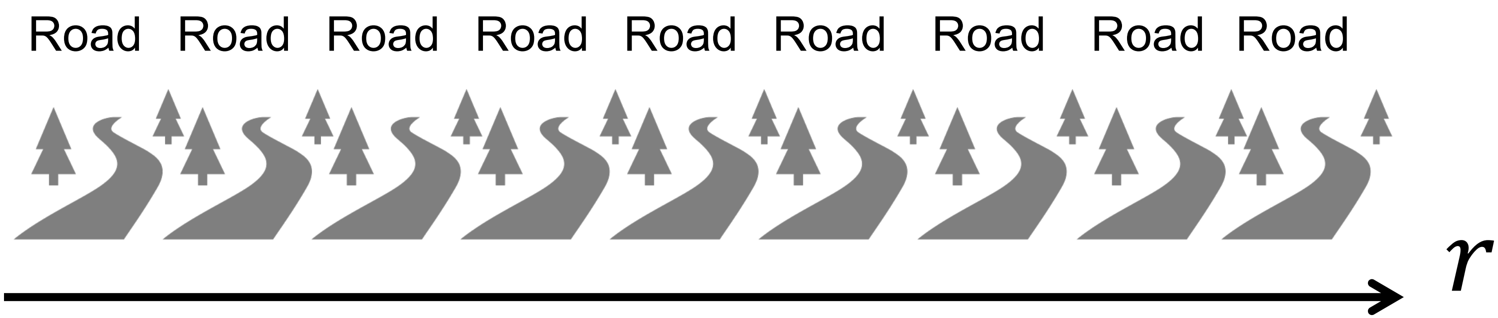
RUTGERS

$\rho_0$ Uniform terrain




# An analogy to energy loss

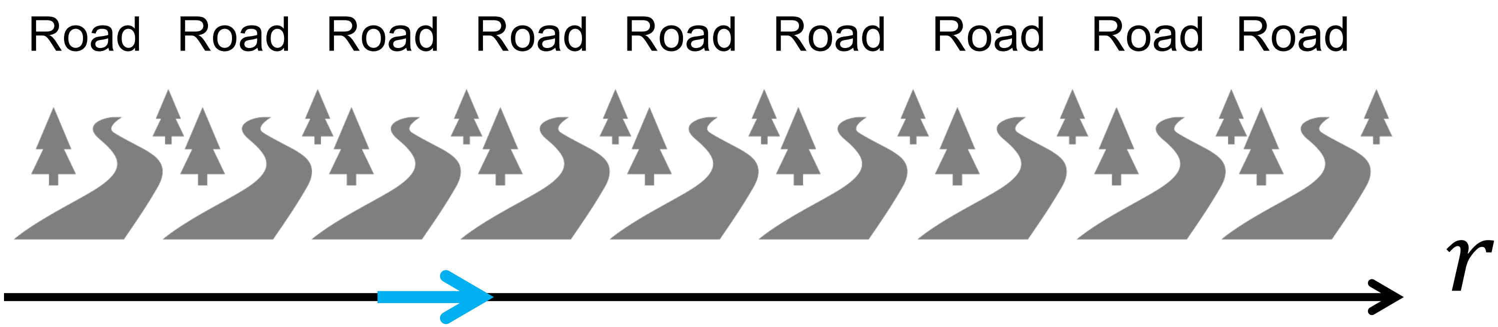
$\rho_0$ Uniform terrain




# The ability to lose energy

RUTGERS

$\rho_0$ Uniform terrain



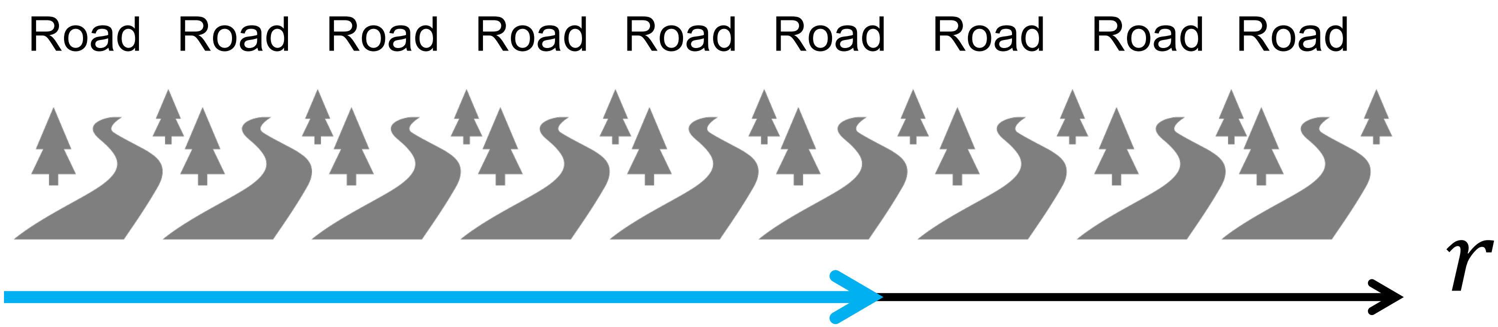

$$S\,\rho_0 \Delta r \propto \Delta E$$

Const. Unit: Energy/distance*

$S$ The ability to (let him) lose energy

# Energy loss in a uniform medium

$\rho$ Uniform terrain

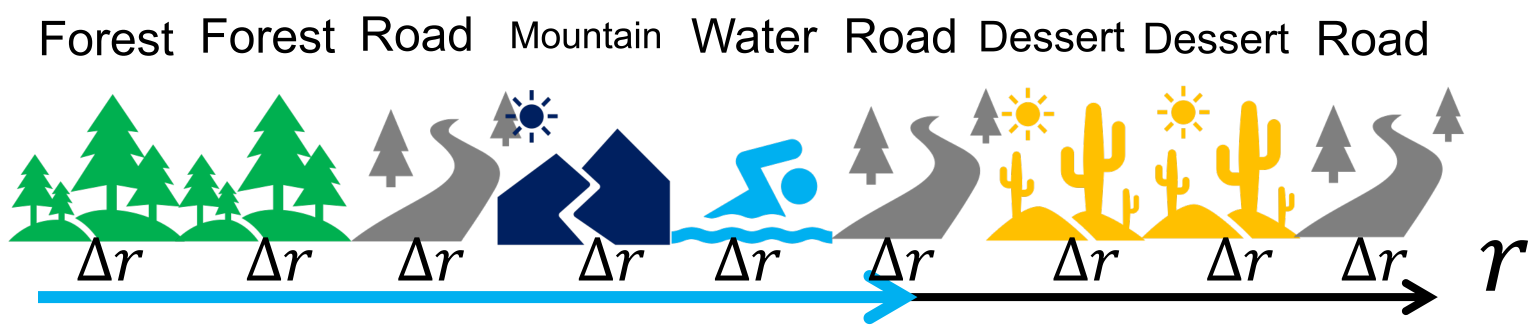



$$S\,\rho L = E_{\text{loss}}$$

$$\propto \rho L$$

# Heterogeneous terrain

$\rho_i$ Terrain




$$\Delta r \sum \rho_i$$

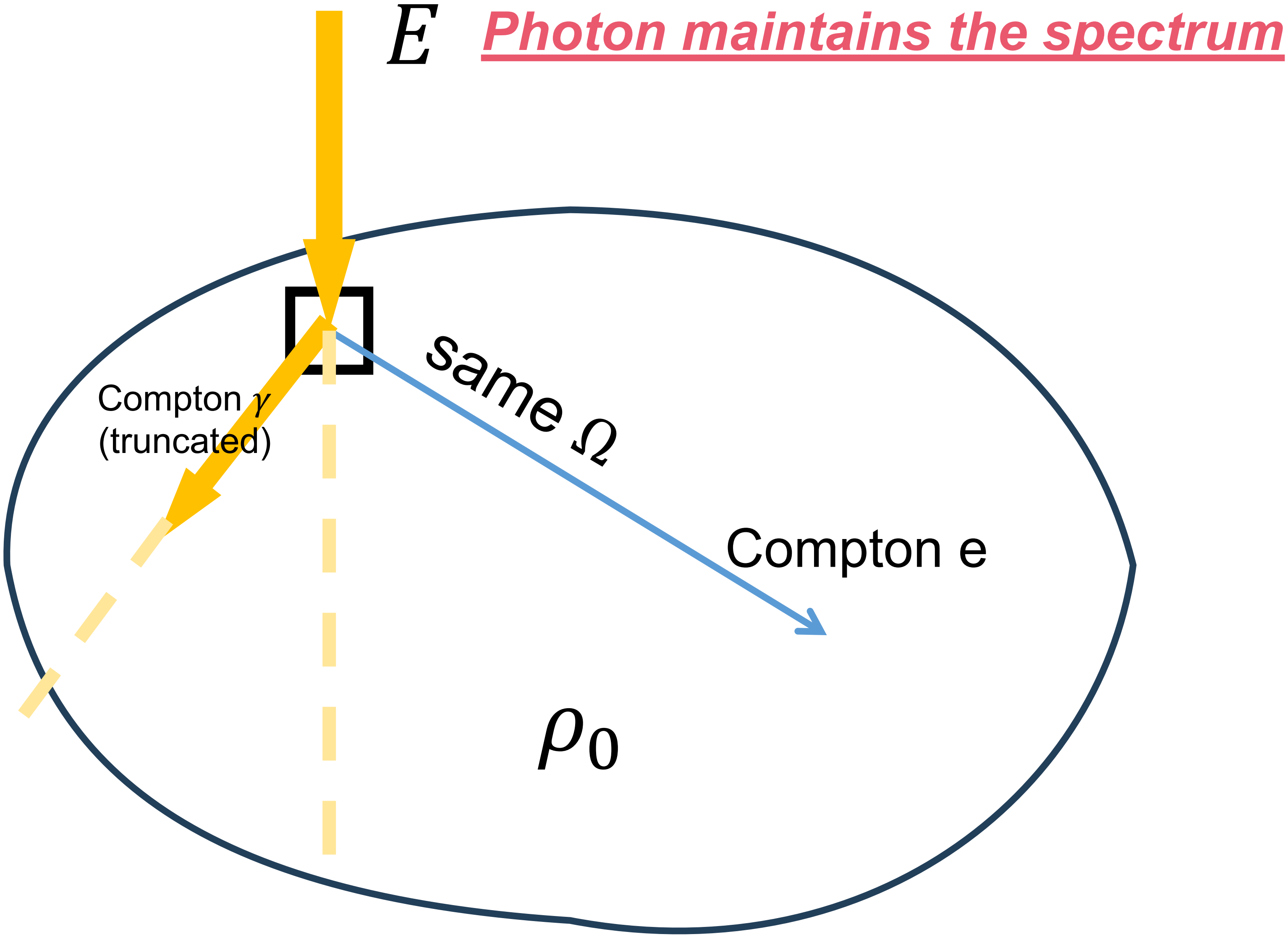
$$= \rho_0 L$$

Same initial energy

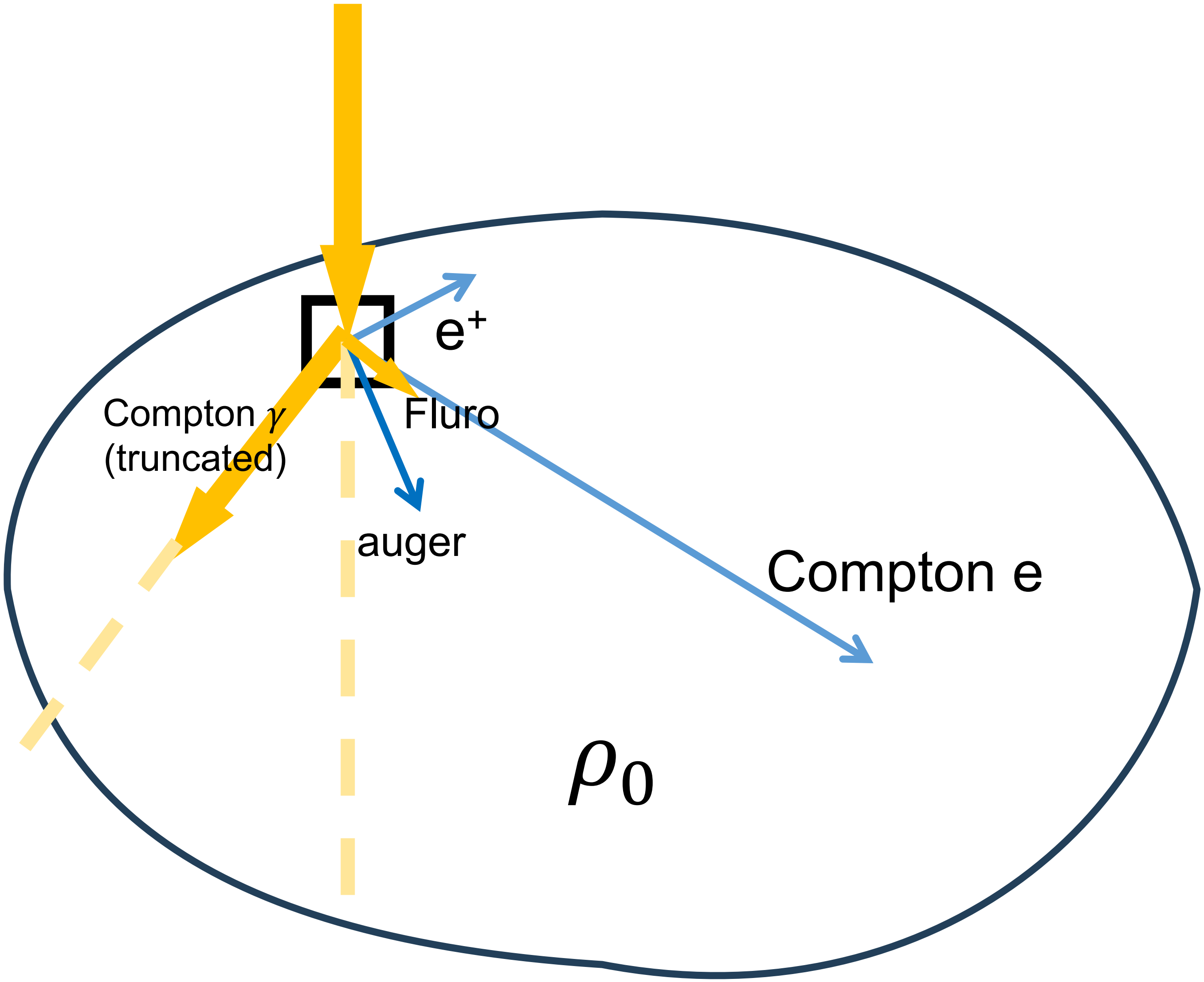
Lose the same energy

# Photon Perspective

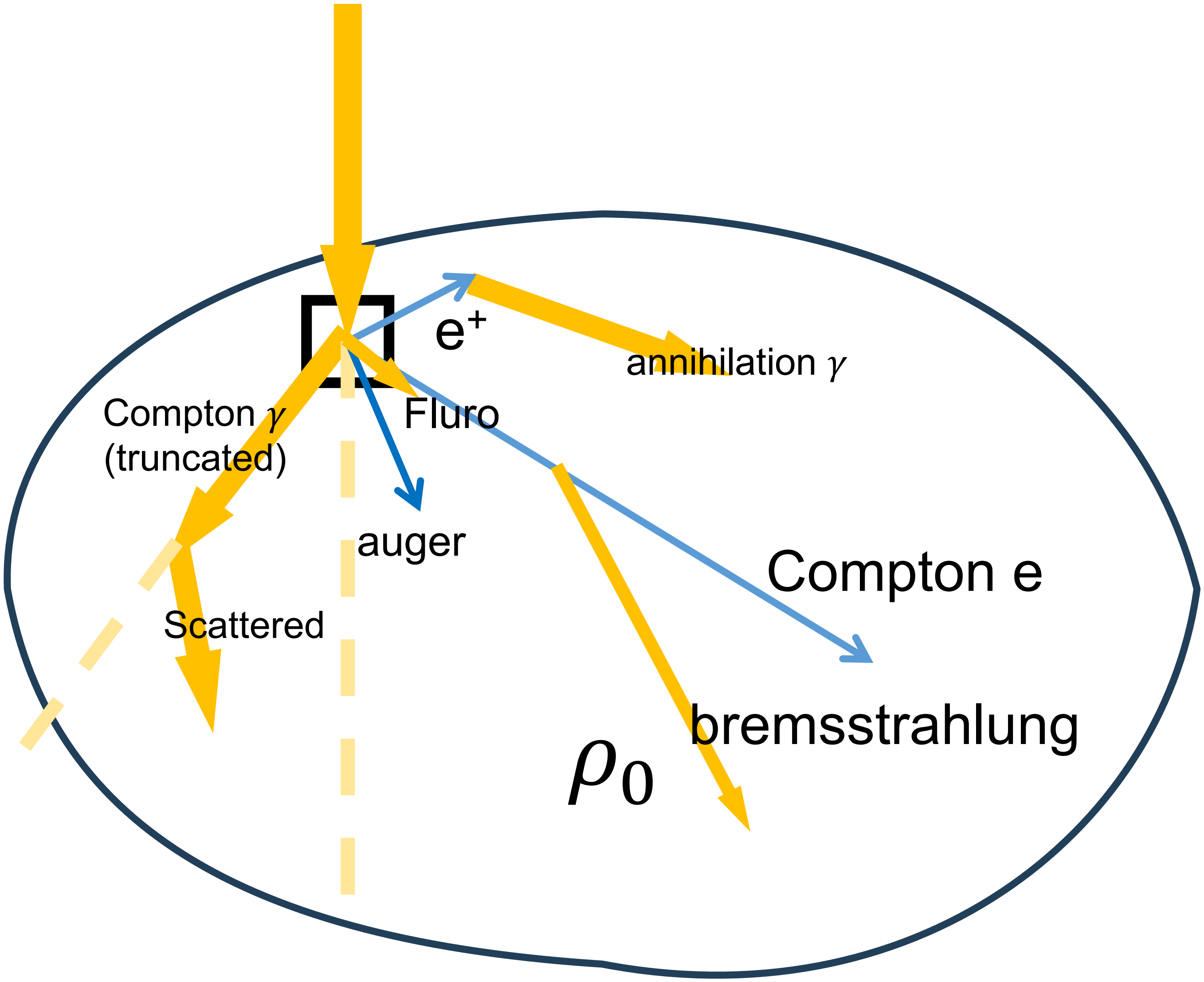
RUTGERS



# Photon Perspective

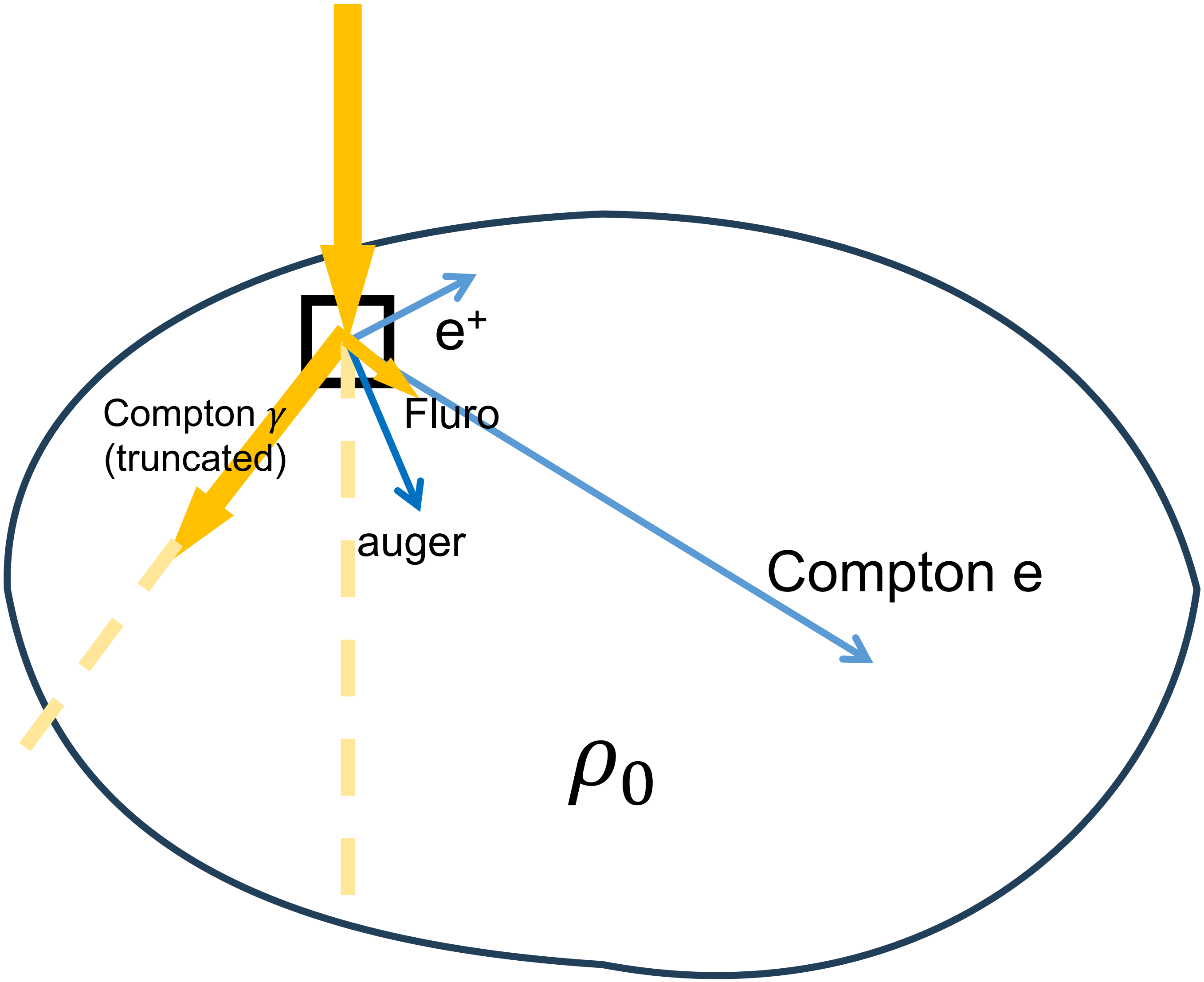


# Photon Perspective (Secondary)



# Photon Perspective (Primary Dose Deposition)

RUTGERS

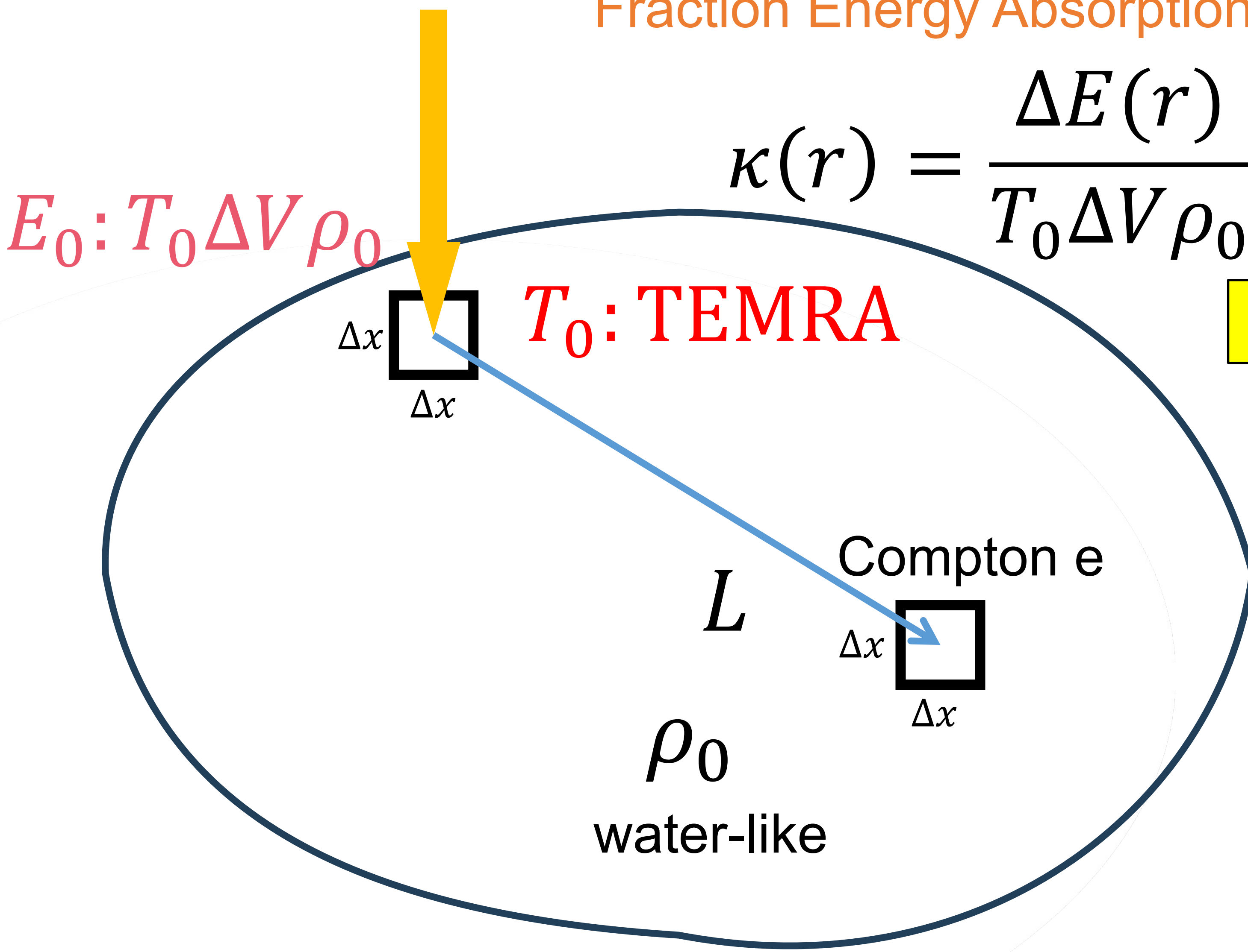


# Electron Perspective

Fraction Energy Absorption Kernel

$$\kappa(r) = \frac{\Delta E(r)}{T_0 \Delta V \rho_0} \propto \frac{e^{-r}}{r}$$

Known



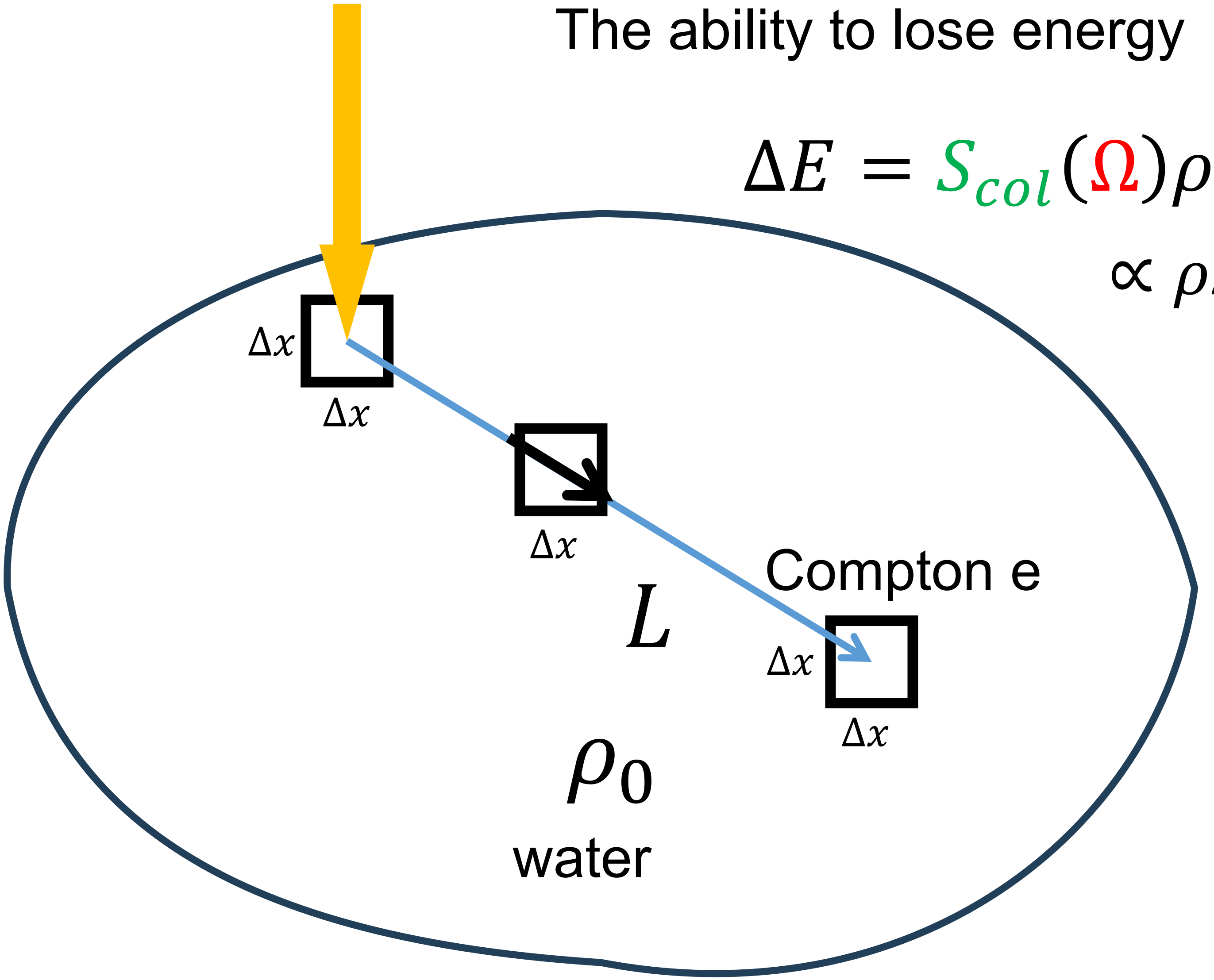
# Electron Perspective

RUTGERS

The ability to lose energy

$$\Delta E = S_{col}(\Omega)\rho_0 \Delta x$$

$$\propto \rho \Delta x$$

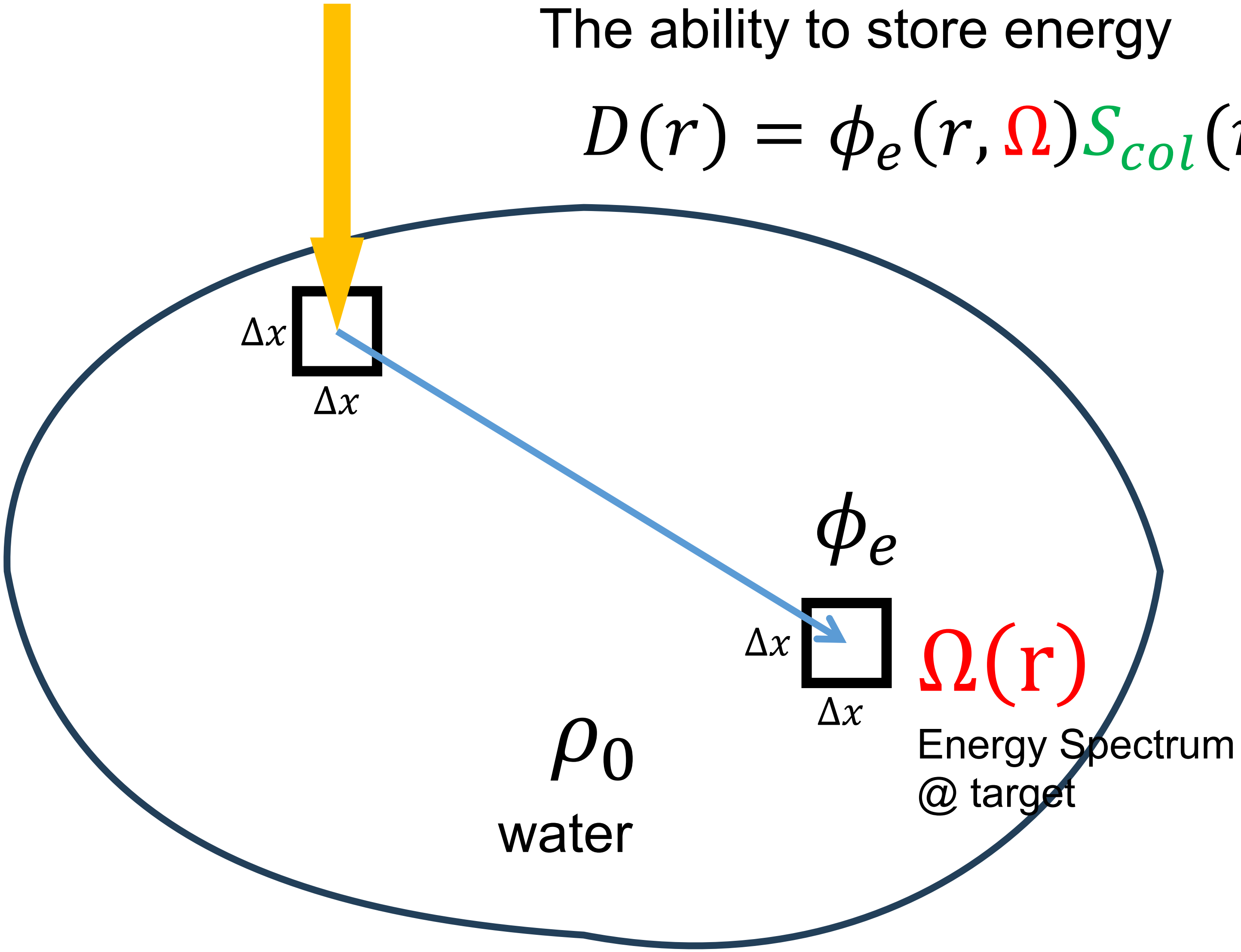


# Electron Perspective

RUTGERS

The ability to store energy

$$D(r) = \phi_e(r, \Omega) S_{col}(r, \Omega)$$

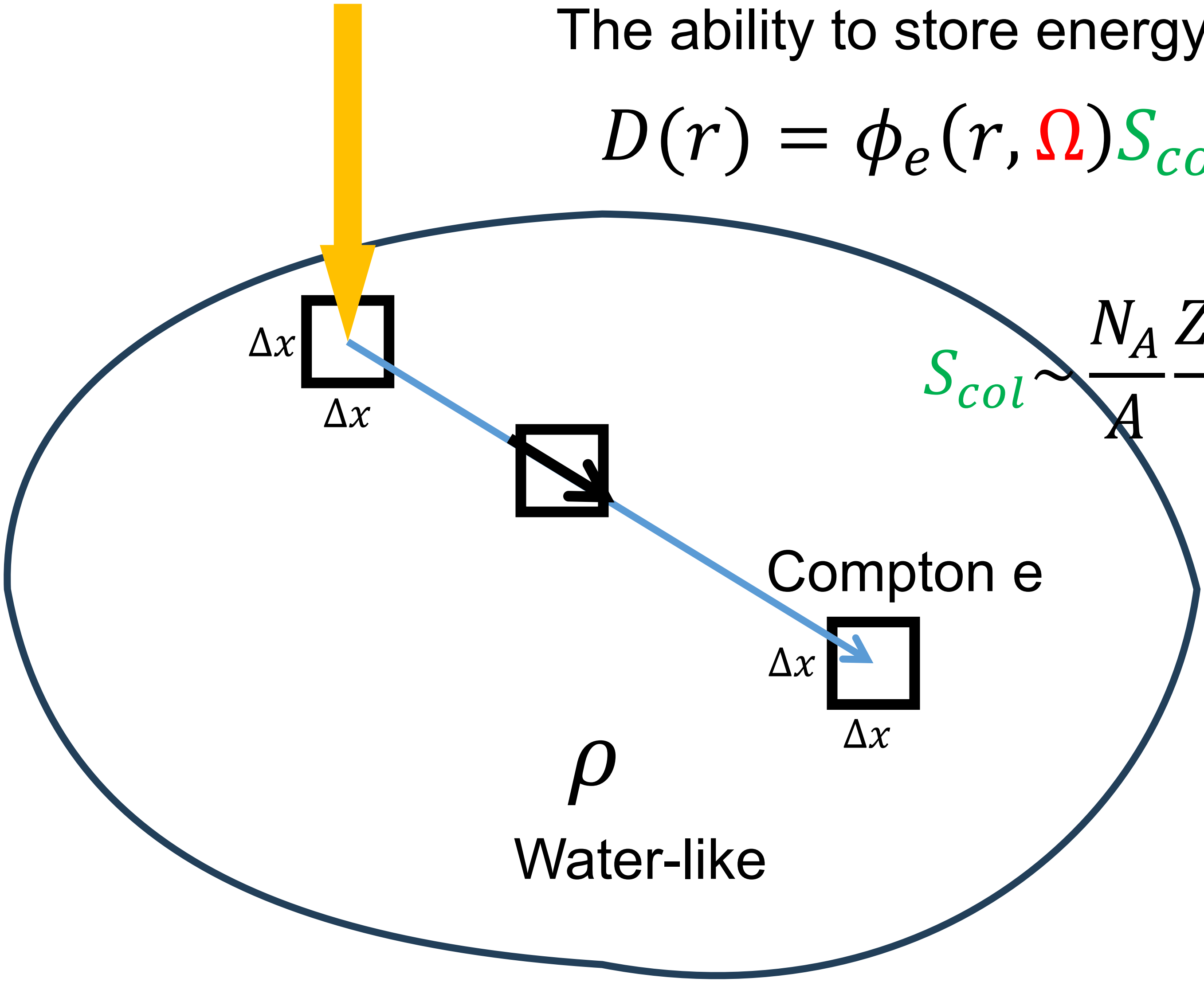


# Electron Perspective

The ability to store energy

$$D(r) = \phi_e(r, \Omega) S_{col}(r, \Omega)$$

$$S_{col} \sim \frac{N_A}{A} \frac{Z f(Z)}{\beta^2}$$

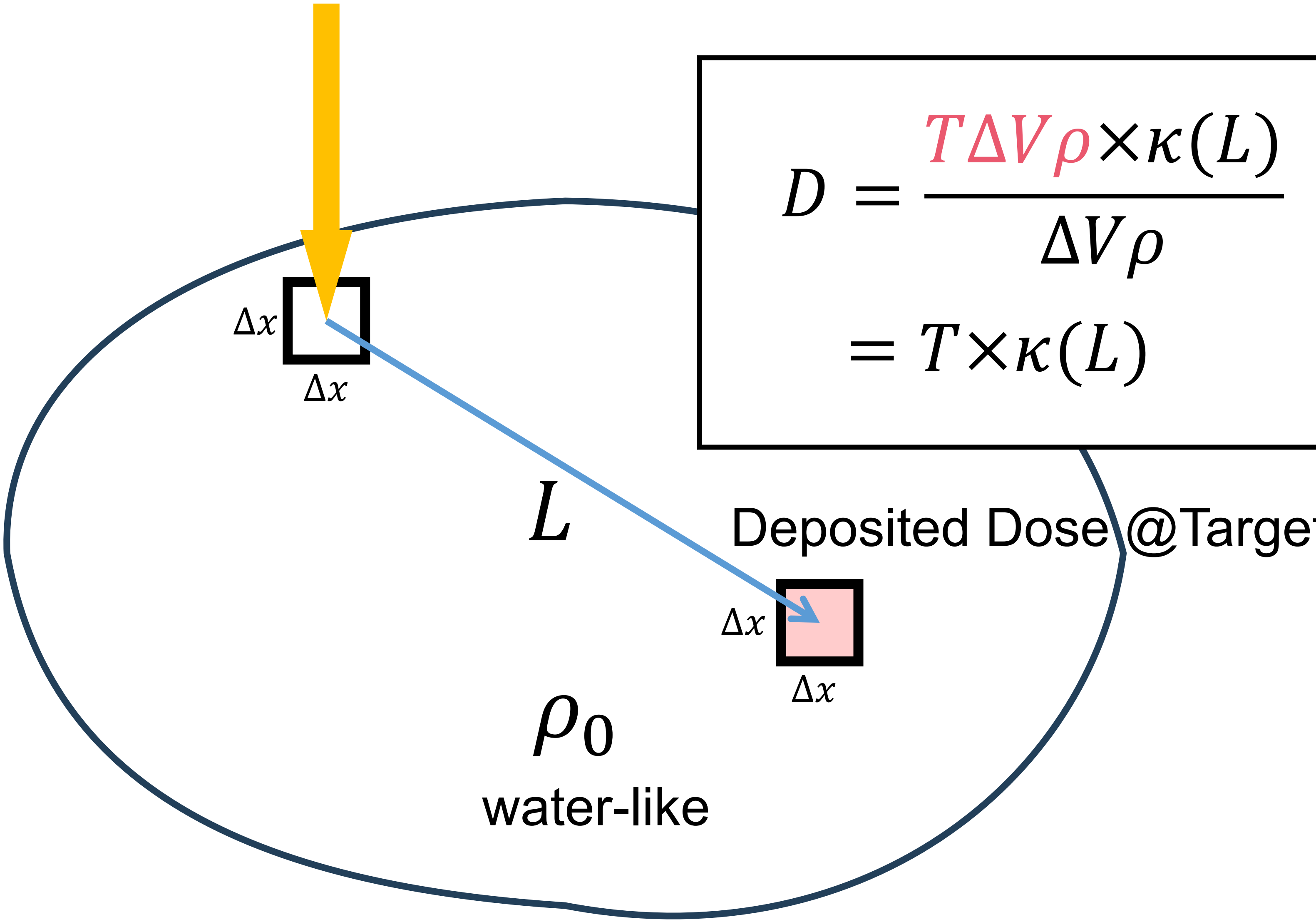


# Dose Deposition

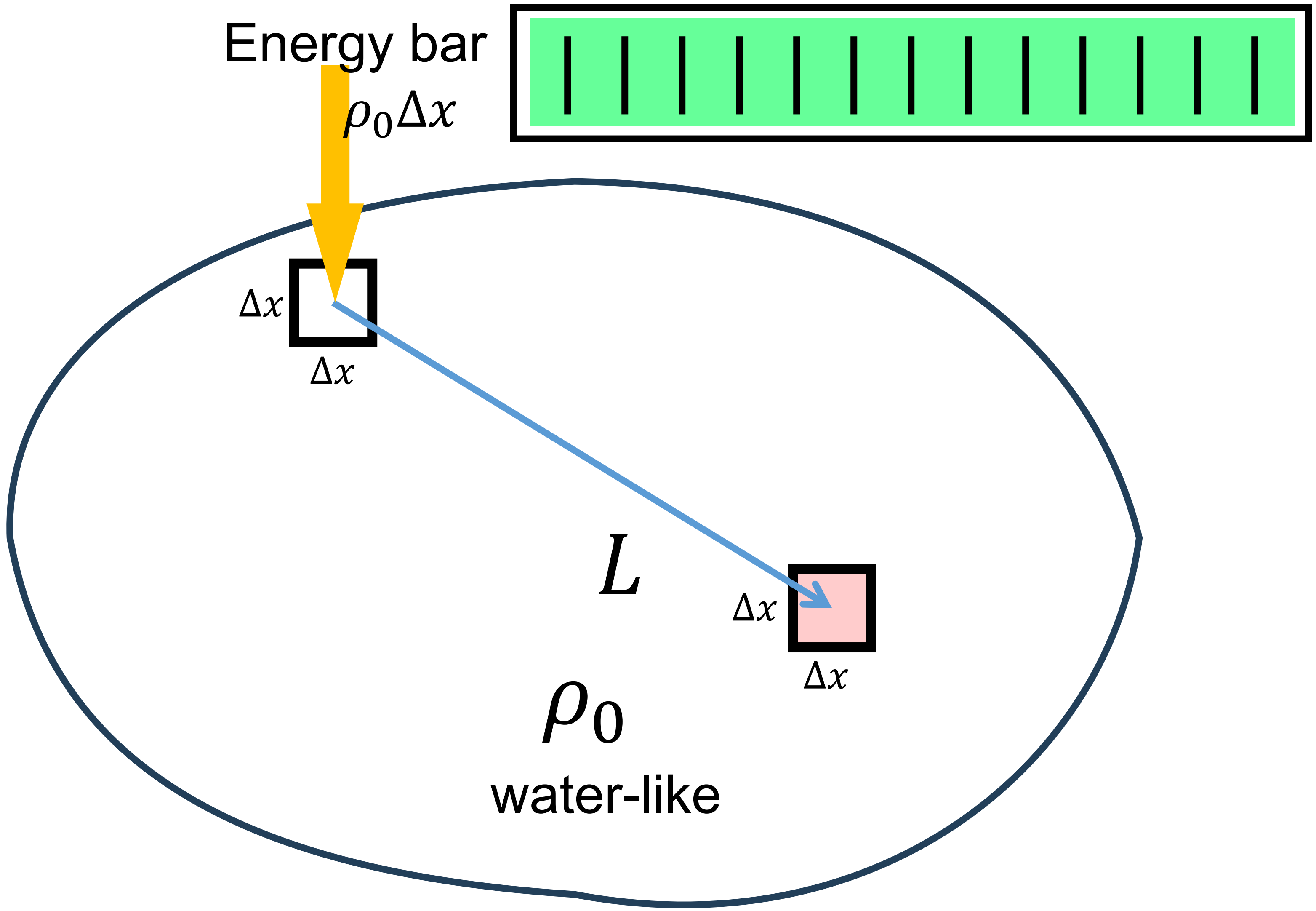


$$D = \frac{T\Delta V\rho \times \kappa(L)}{\Delta V\rho}$$

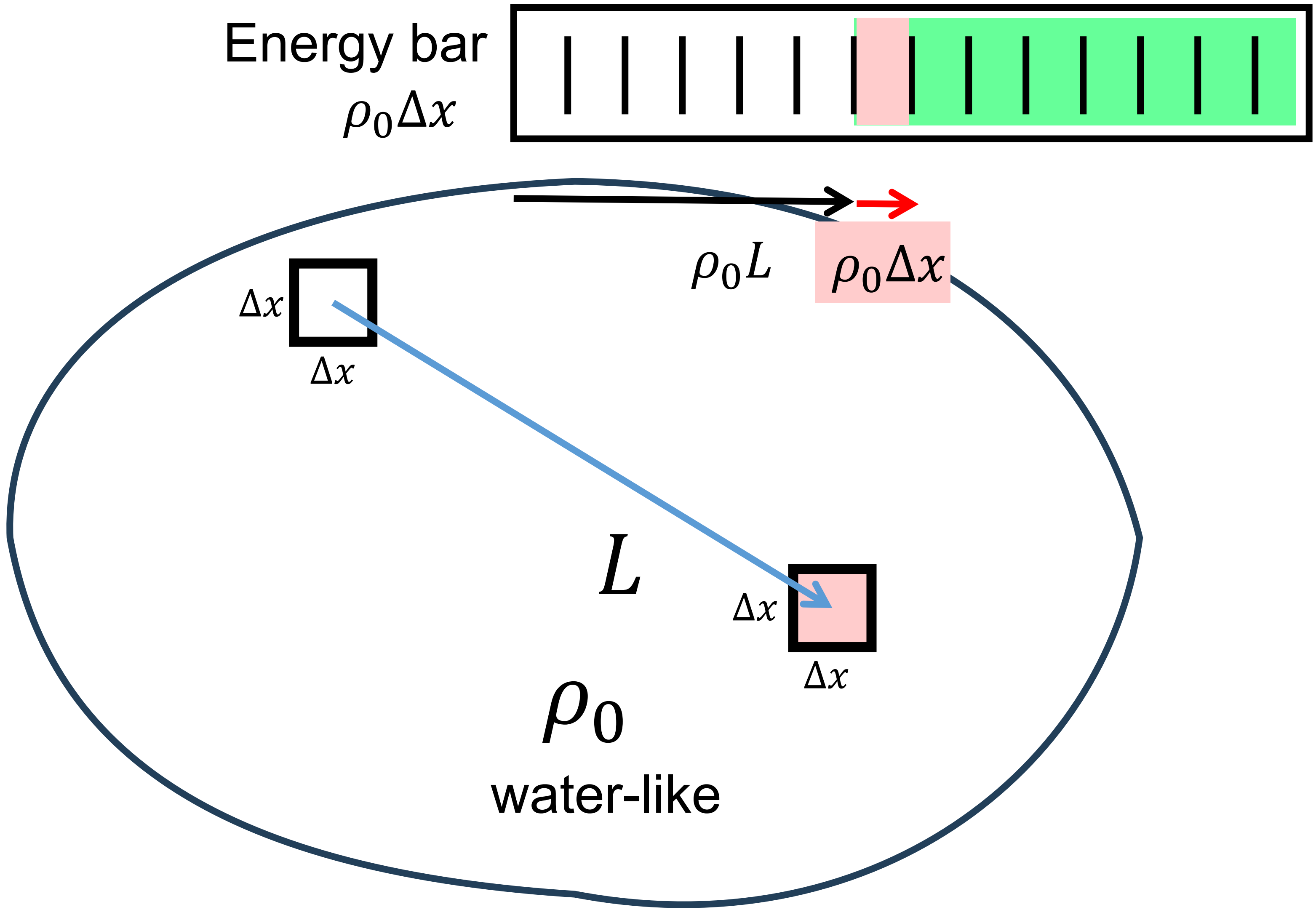
$$= T \times \kappa(L)$$



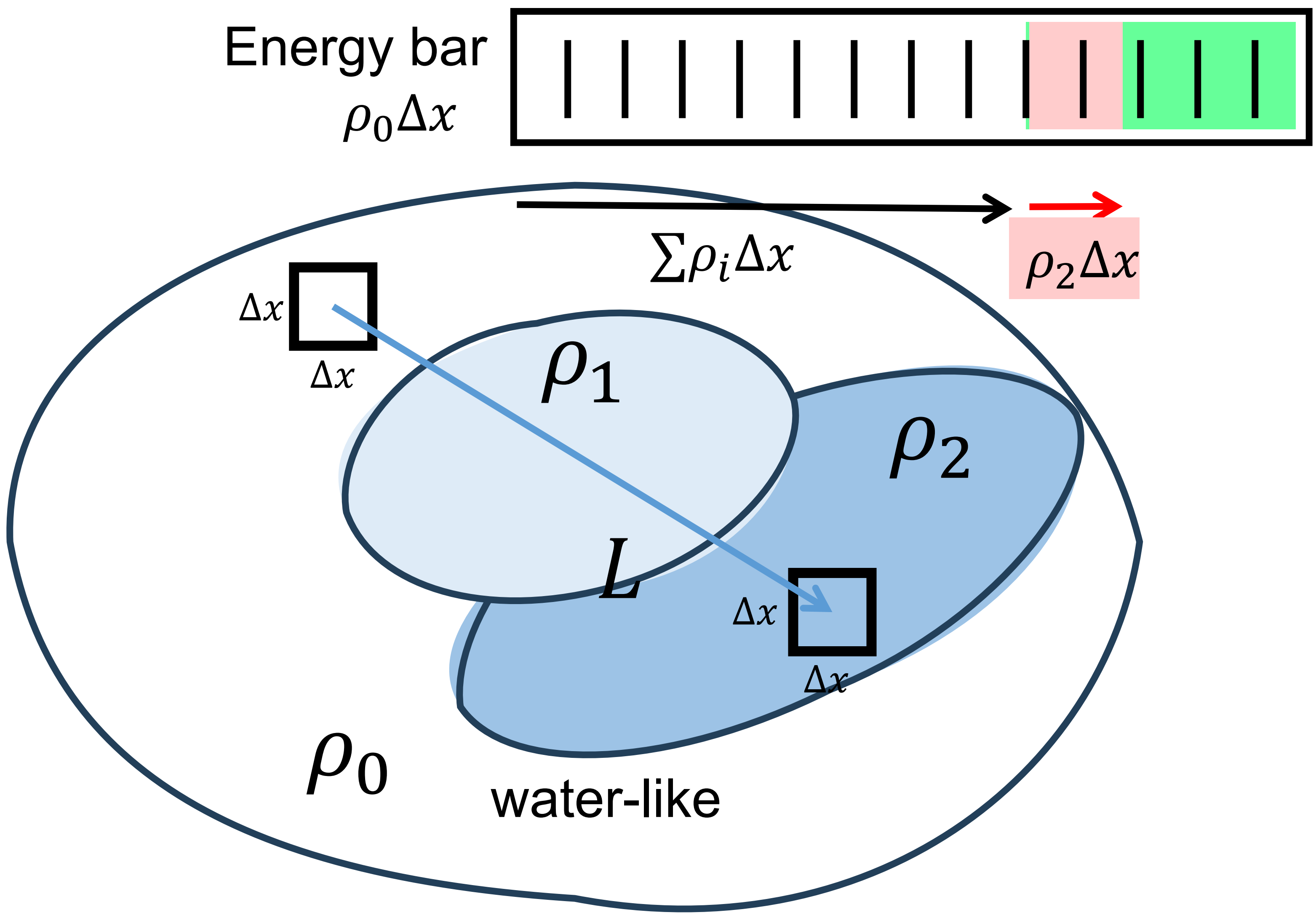
# Dose Deposition



# Dose Deposition

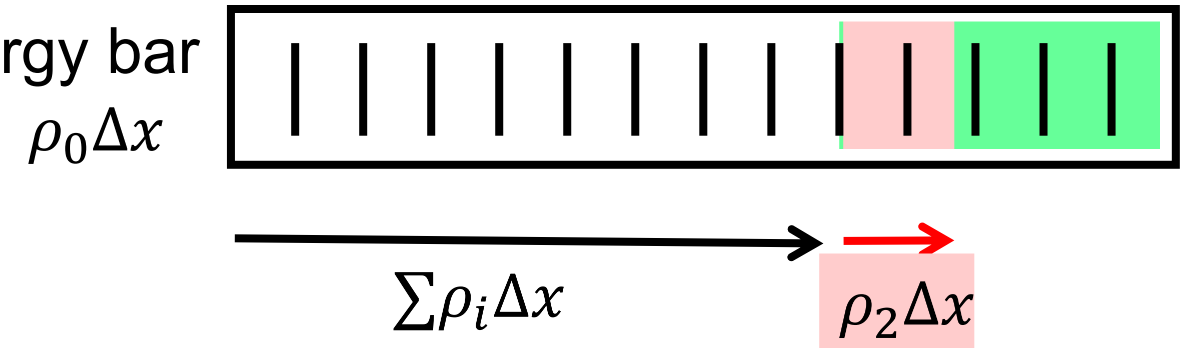


# Heterogeneity Correction



# Dose Deposition (Corrected)

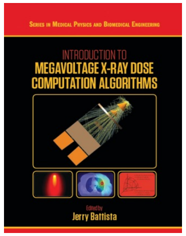
Energy bar




Starting pixel — original dose — Eneryg loss — Energy @target

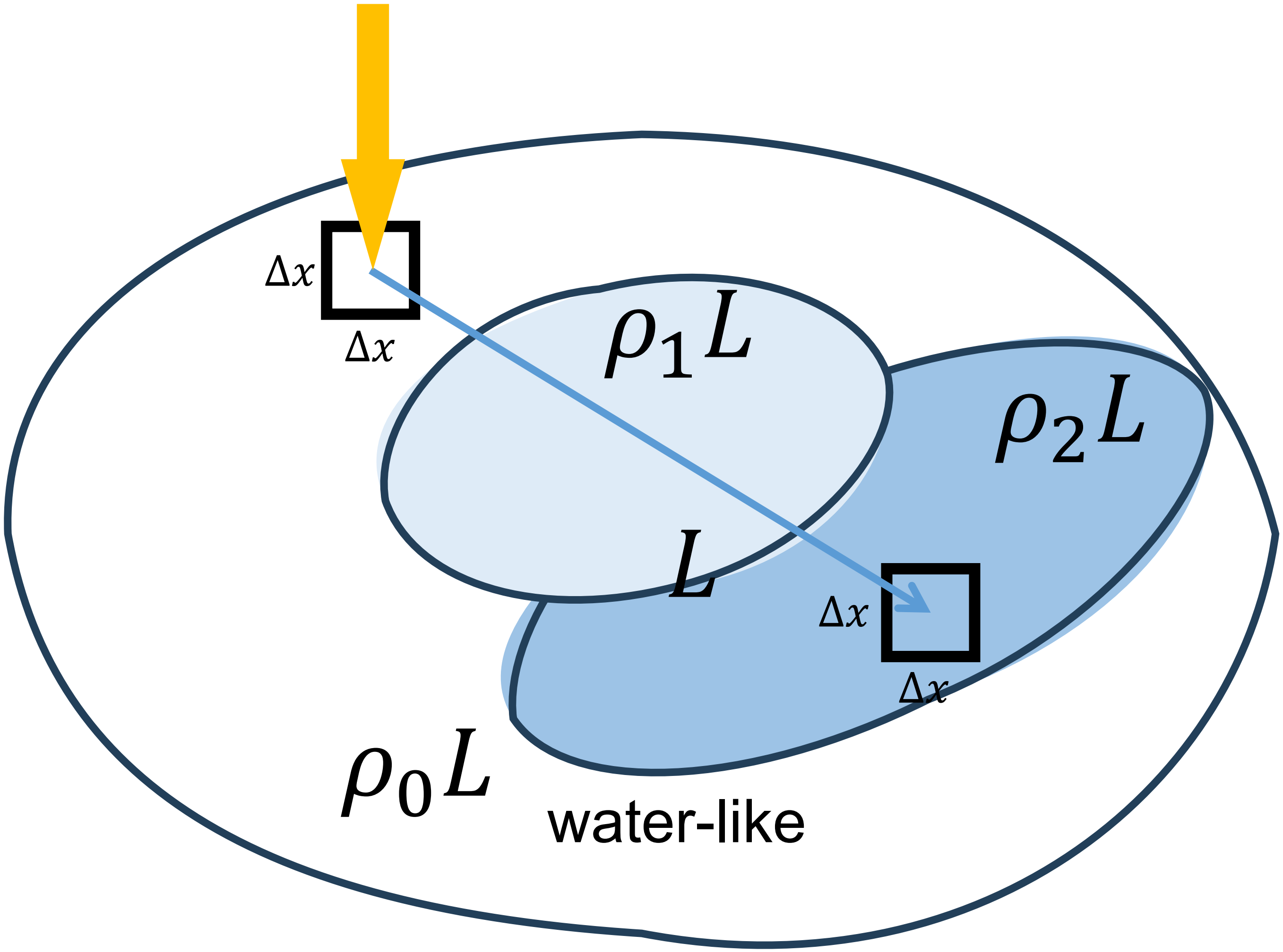
$$D(\text{target}) = \frac{aT\Delta V\rho_0 \times \kappa\left(\frac{\sum \rho_i}{\rho_0} L\right) \times \frac{\rho_2}{\rho_0}}{a\Delta V\rho_2}$$

$$= T \times \kappa\left(\frac{\sum \rho_i}{\rho_0} L\right)$$

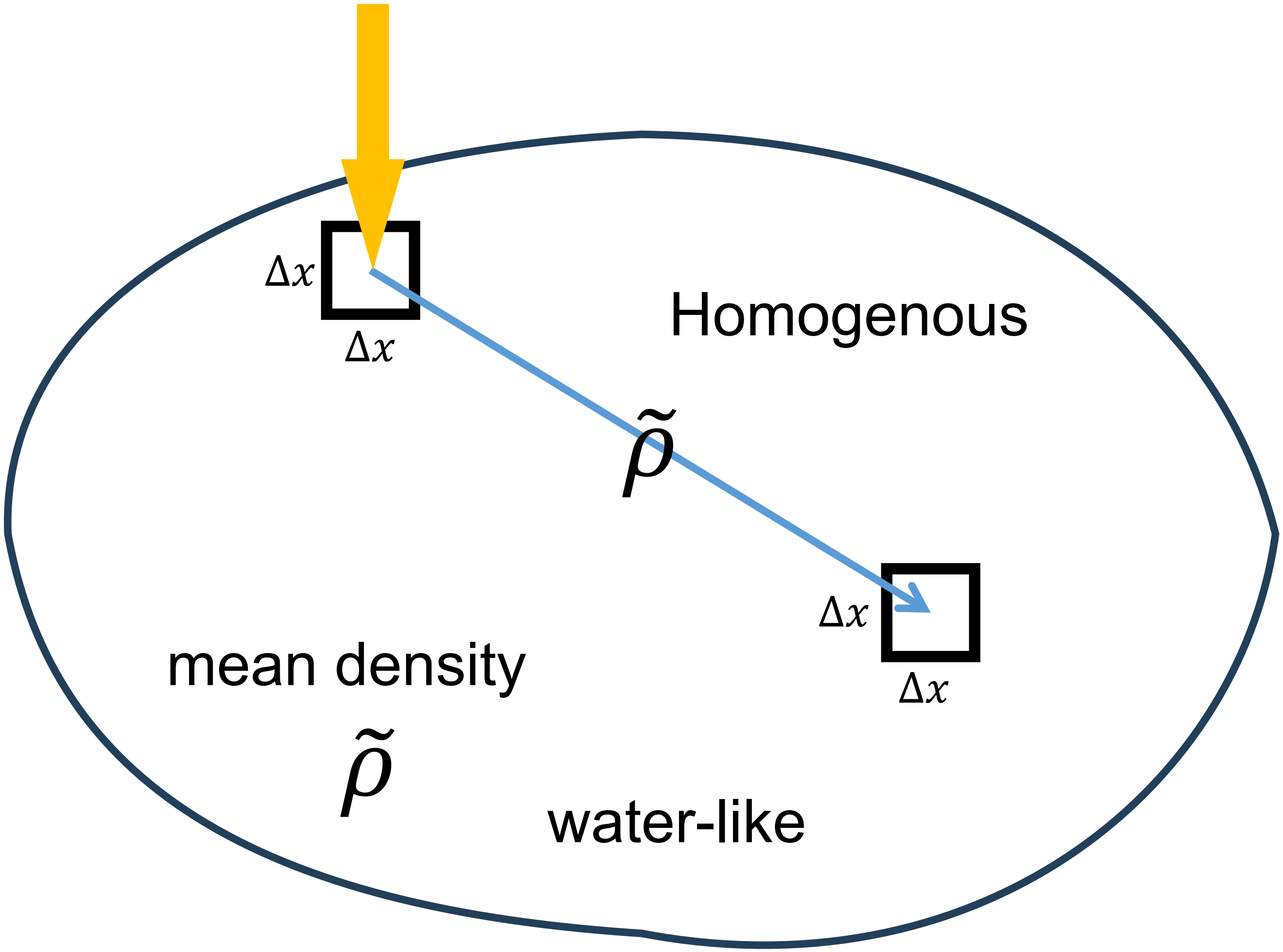



*This result is also obtained by Battista, et al. w/o divergence correction

# Mackie's method



# Mackie's method



# Some speculation

RUTGERS

- Just to simplify the physics picture?
- Computation constraint back in 1985?


Robert Wood Johnson University Hospital | RWJBarnabas HEALTH

Contact: cz453@cinj.Rutgers.edu