Edited by Jong Chul Ye, Yonina C. Eldar, and Michael Unser

Deep Learning for Biomedical Image Reconstruction

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Deep Learning for Biomedical Image Reconstruction

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6 Deep Learning in CT Reconstruction: Bringing the Measured Data to Tasks

Guang-Hong Chen, Chengzhu Zhang, Yinsheng Li, Yoseob Han, and Jong Chul Ye

6.1 Introduction

The positive impact that diagnostic computed tomography (CT) imaging has made upon the diagnosis and treatment of symptomatic patients over the past few decades is unprecedented. However, despite the remarkable contribution of CT to modern healthcare, the issue of the small theoretical cancer risk associated with the use of ionizing radiation in CT has recently become a public concern.

To address the public concerns, many strategic discussion sessions have been organized by the National Institutes of Health (NIH) and the American Association of Physicists in Medicine (AAPM) to map out the scientific steps needed to accomplish the goal of sub-mSv CT imaging. A consensus report was published [1] to guide the development of low-dose CT imaging technologies. The research and development efforts from academic, industrial, and clinical societies in the past decade have resulted in the development and implementation of a variety of radiation-dose-reduction strategies for CT exams.

To lower the radiation dose in CT exams, advanced CT hardware technologies have been developed and incorporated into the current clinical CT systems, including but not limited to, newer generations of CT detectors with improved detection quantum efficiency, beam-shaping filters and dynamic *z*-axis collimators that limit the dose at the periphery of the patient and edge of the scan range, automatic exposure control with the flexibility to modulate tube potential and tube current, and photoncounting CT.

In addition to hardware technology developments, software technologies have also played a major role in this joint effort of radiation dose reduction. Many advanced image-processing techniques working in the sinogram projection-data domain or in the reconstructed-image domain, or in both domains jointly in fully statistical iterative reconstruction (IR) format have been developed and some of these methods have been incorporated into commercial products for routine clinical uses. In the past decade or so, such newly developed low-dose reconstruction or processing software has been rigorously evaluated with large patient cohorts. It has been found that, after a decade-long effort to reduce CT radiation dose with IR software techniques, the radiation dose can be reduced by up to 25 percent with statistical IR techniques *without sacrificing clinical diagnostic quality* [2, 3].

This status quo of the radiation dose reduction factor using statistical IR leaves plenty of room for the development of new techniques to further reduce radiation dose in clinical CT exams, particularly, new techniques that are sufficiently flexible to leverage the intrinsic advantages of analytical reconstruction methods, such as filtered back projection or differentiated back projection filtration-based methods [4] and the statistical learning principles that have been extensively investigated in statistical IR methods. In this regard, the recent advances in deep learning provide us with an ideal computational framework that can be developed to directly reconstruct CT images from sinogram data, or can be combined with analytical reconstruction methods to address the challenges encountered in their application, or can be combined with statistical IR to address reconstruction accuracy and the generalizability concerns associated with deep-learning methods.

The foundational physics principles of CT image reconstruction problems will be presented in Section 6.2. After that, deep learning in CT image reconstruction will be described, with examples on how to perform reconstruction directly from sinogram to image in Section 6.3, how to combine deep learning with analytical reconstruction methods in Section 6.4, and how to combine deep learning with statistical IR methods in Section 6.5.

6.2 CT Imaging Physics and Reconstruction-Problem Formulations

In this section, by following the experimental image-formation processes in CT, image-reconstruction problems are formulated from an imaging-physics standpoint in order to make the scientific foundations of each reconstruction framework transparent. Particularly, the assumptions implicitly or explicitly used in the past will be made transparent in our formulations. This review of problem formulation also aims to put the rapid rising of deep-learning methods into a proper scientific context and to find a niche for deep-learning methods in the big picture of radiation dose reduction in CT. We also hope that this section will inspire cross-pollination between analytical image reconstruction, statistical IR, and data-driven deep-learning-based reconstruction strategies, with the ultimate objective of developing some innovative reconstruction strategies with a high clinical impact.

6.2.1 CT Image Reconstruction Problem Formulation: A Deterministic Approach

The physics of X-ray attenuation by an image object is captured by the empirical Beer– Lambert law, which characterizes the mean photon number before and after the X-ray beam interacts with the image object. The interactions of the X-ray photons and the image object are characterized by the so-called linear attenuation coefficient, $\mu(\vec{x}, \varepsilon)$, at a given spatial location and X-ray energy ε . The mean photon number recorded at the *i*th detector element is given by the Beer–Lambert law as follows:

$$\bar{N}_i = \bar{N}_{i0} \int_0^{\varepsilon_{max}} d\varepsilon \Omega(\varepsilon) \exp\left(-\int_{\ell_i} d\vec{x} \,\mu(\vec{x},\varepsilon)\right),\tag{6.1}$$

Therefore, when the repetition of experimental data acquisition is prohibited, as in clinical CT acquisitions, *CT image reconstruction must be formulated as a statistical learning problem: to statistically infer the object information* $\mu(\vec{x}, \bar{\epsilon})$, *i.e., the statistical parameters given a set of measured data* $\{N_1, N_2, \ldots, N_M\}$.

Since the measurements at different detector elements can be considered to be statistically independent, the joint probability of a measured dataset is then given by

$$P(\{N_1, N_2, \dots, N_M\} | \mu(\vec{x})) = \prod_{i=1}^M P(N_i | \mu(\vec{x})) = \prod_{i=1}^M \frac{\bar{N}_i^{N_i} e^{-\bar{N}_i}}{N_i!}, \quad (6.5)$$

where the effective energy dependence $\bar{\varepsilon}$ in $\mu(\vec{x}, \bar{\varepsilon})$ has been omitted, to avoid notational cluttering. When the statistical parameters, i.e., the mean counts, are known for each measurement, the above equation dictates the joint probability for the set of measurements. Now the CT reconstruction problem can be formulated as a standard statistical learning problem: after the measurements $\{N_1, N_2, \ldots, N_M\}$ are given, how does one estimate the statistical parameters $\{\bar{N}_1, \bar{N}_2, \ldots, \bar{N}_M\}$? Or, alternatively, how does one estimate $\mu(\vec{x})$ since these linear attenuation coefficients are related to the mean counts as shown in Eq. (6.3)? There are several different approaches in statistics to address this statistical learning problem, as will be discussed below.

Maximum Likelihood Method

The first way of addressing the above statistical learning problem is to use Fisher's maximum likelihood (ML) method. In this method, when the values of $\{N_1, N_2, ..., N_M\}$ are obtained from experimental measurements, the formula on the right-hand side of Eq. (6.5) becomes a function of the statistical parameters $\{\bar{N}_1, \bar{N}_2, ..., \bar{N}_M\}$ and this function is called the likelihood of the statistical parameters. To find a point estimate of the statistical parameters, what Fisher suggested is to maximize the log-likelihood function as follows:

$$\hat{\mu}(\vec{x}) = \arg \max_{\mu(\vec{x})} \sum_{i=1}^{M} (N_i \ln \bar{N}_i - \bar{N}_i),$$

= $\arg \max_{\mu(\vec{x})} \sum_{i=1}^{M} \left(-N_i \int_{\ell_i} d\vec{x} \, \mu(\vec{x}) - \bar{N}_{i0} \exp\left(-\int_{\ell_i} d\vec{x} \, \mu(\vec{x})\right) \right)$ (6.6)

where \bar{N}_i in Eq. (6.2) has been substituted in the second line to make the target quantity $\mu(\vec{x})$ transparent. The term that is not related to \bar{N}_i has been dropped in the above equation. If we denote $P_i = \int_{\ell_i} d\vec{x} \mu(\vec{x})$, then the above optimization problem enjoys the following stationarity conditions:

$$P_i = \int_{\ell_i} d\vec{x} \,\mu(\vec{x}) = \ln \frac{\bar{N}_{i0}}{N_i}, \qquad i = 1, 2, \dots, M.$$
(6.7)

Therefore, one can solve for $\mu(\vec{x})$ using the measured quantities \bar{N}_{i0} and N_i . In this ML estimate formulation, repeated measurements are required to infer \bar{N}_{i0} , but this can be done experimentally since there is no image object involved and thus there is no concern of radiation risks to a patient.